

DETAILS EXPLANATIONS**[PART : A]**

1. For any closed surface net outgoing electric flux is equal to total charge enclosed by that surface this surface is called Gaussing surface.

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{enc}$$

$$\oint_S \vec{D} \cdot \vec{ds} = \int_V (\vec{\nabla} \cdot \vec{D}) dv = \int_V \rho_v dv$$

$$\vec{\nabla} \cdot \vec{D} = \rho_v$$

2. Poisson equation

$$\nabla^2 V = \frac{\rho_v}{\epsilon}$$

$V \rightarrow$ Voltage (Potential) field

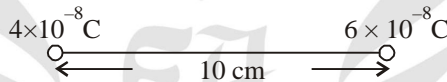
$\rho_v \rightarrow$ Volume charge density

Laplace equation

if $\rho_v = 0$

$$\nabla^2 V = 0$$

- 3.



$$F = \frac{q_1 \times q_2}{4\pi \epsilon_0 d^2}$$

$$= \frac{4 \times 10^{-8} \times 6 \times 10^{-6} \times 9 \times 10^9}{0.1 \times 0.1}$$

$$= 24 \times 9 \times 10^{-3} \text{ N.}$$

4. Surge impedance of a transmission line is the characteristic/intrinsic impedance of line. It is the ratio of forward max. Electric field to the forward max magnetic field

$$Z_0 = \frac{E_0^+}{H_0^+} = \left(\sqrt{\frac{R + j\omega L}{G + j\omega c}} \right)$$

It is not depend on the length of line.

$$5. \quad \bar{\nabla} \times \bar{E} = \mu \frac{d\bar{H}}{dt} \quad \dots(1)$$

$$\bar{\nabla} \times \bar{H} = \epsilon \bar{E} + \frac{d}{dt} \bar{E} \quad \dots(2)$$

$$\bar{\nabla} \cdot \bar{D} = \rho_v \quad \dots(3)$$

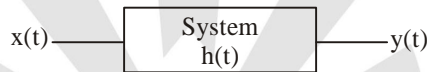
$$\bar{\nabla} \cdot \bar{H} = 0 \quad \dots(4)$$

Where $\bar{D} = \epsilon \bar{E}$

6. A linear and time invariant system can be described by input-output relationship or impulse response.

I. If LTI system is represented by I/P –O/P relationship

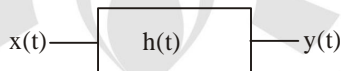
Let a LTI system which have input $x(t)$ and output $y(t)$



LTI system is stable if input is bounded then output must be bounded for all the instant of time.

If LTI system is represented by impulse response

Let impulse response of system is $h(t)$ then



The given system is stable then

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

The impulse response should be finite.

7. Continuous wave radar is a type of radar system, Where a known stable frequency continuous wave radio energy is transmitted and then received from any reflecting object.

Continuous radar uses Dopplers, which render the radar immune to interference from large stationary objects and slow moving clutter. Continuous radar system are used at both ends of the range spectrum.

There are two types of continuous wave radar

1. Unmodulated continuous wave :

In this the return frequencies are shifted away from the transmitted frequency based on the Doppler effect when objects are moving. There is no way to evaluate distance.

2. Modulated continuous wave :

Frequency modulated continuous wave radar (FMCW) also called continuous wave frequency modulated (CWFM) radar.

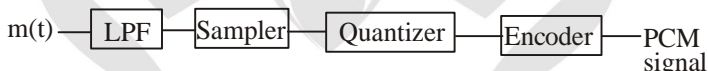
It is a short range measuring radar set capable of determining distance. This increases reliability by providing distance measurements along with speed measurement, which is essential when there is more than one source of reflection arriving at the radar antenna.

Advantages :

1. Radar are inexpensive, simple, fully automated and cheap to maintain.
2. CW radar can detect accurately upto 100 km distance.
3. Without increasing power number of receiver sample can be increased or decreased.

8. Pulse code modulation is a conversion method from analog nature to digital nature. In pulse code modulation, the continuous signal is passed through a sampler and quantizer. Sampler provide discretization on time axis and quantization process provide discretization on amplitude axis.

Hence resultant signal is a digital signal.



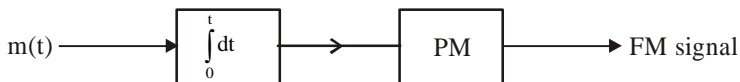
Pulse modulation system is a modulation process in which the high frequency carrier is a rectangular pulse train. The one of the parameter of this carrier is varied according to message signal.

In pulse modulation system the parameter is only varying according to message signal but in PCM it is discretized on time and amplitude axis. Hence PCM is a digital techniques.

9. (i) In FM, the information of signal is encoded in its frequency. Similarly for PM, information is encoded in its phase.
 (ii) Frequency deviation in FM do not depends on modulating signal frequency, while in PM frequency deviation depends on input signal frequency.

$$\text{FM signal } X_{\text{FM}}(t) = A_c \cos\left(\omega_c t + k_f \int m(t) dt\right)$$

$$\text{PM signal } X_{\text{PM}}(t) = A_c \cos(\omega_c t + k_p m(t))$$



10. Gaussian process or normal process :

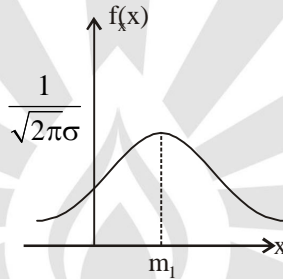
A Gaussian process $f(x)$ is a collection of random variables, any finite number of which have Gaussian Distribution.

Gaussian Distribution

For a Gaussian random variable x , with mean m_1 and variance σ^2 the probability distribution is given by pdf.

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{(x-m_1)^2}{2\sigma^2}\right)}$$

Gaussian process is used to describe a set of Gaussian random variable.



11. The diodes which have low reverse recovery time, of about $5 \mu\text{s}$ or less, are classified as fast recovery diodes. These are used in chopper, commutation circuits, SMPS, induction heating etc. their current rating vary from about 1A to several thousand ampere and voltage rating from 50 V to about 3 kV for voltage rating below about 400 V, the epitaxial process is used for diode fabrication and have fast recovery time as low as 50 ns.

12. The value of $\frac{di}{dt}$ can be maintained below acceptable limit by using

a small inductor, called $\frac{di}{dt}$ inductor, in series with the anode circuit

typical $\frac{di}{dt}$ limit value of SCR are 20 – 500 A/ μ sec.

Local slot heating can also be avoided by ensuring that the conduction spreads to the whole area as rapidly as possible this can be achieved by applying a gate current nearer to (but never greater than) the maximum specified gate current.

13. This class of diodes use metal to semiconductor junction for rectification purpose instead of P-N junction. The metal is usually (aluminium) and semiconductor is silicon. Therefore, a schottky diode has aluminium-silicon junction. The silicon is n-type. As compared to P-N junction diode, a schottky diode has

- (i) Lower cut-in voltage
- (ii) Higher reverse leakage current
- (iii) Higher operating frequency

14. GTO is a four layer, three terminal current controlled minority carrier device. It can be turn-on by applying a positive gate current pulse when it is forward biased and turned off by applying a negative gate current.

The GTO gate drive unit should be capable of injecting large positive and negative gate currents with large rate of rise for satisfactory switching of the device. GTOS have relatively low turn off current gain and have $I_L = 2$ Amp.

15. A TRIAC is a bidirectional thyristor with three terminals and five layers. It can conduct from anode to cathode and from cathode to anode. It's operating in four modes so it can be used as AC switch. Most suitable when supply voltage is low frequency AC. It is only preferred for low and medium power applications. TRIAC requires more current for turn-on than SCR at a particular voltage.

16. The fourier series for source current is I_s

$$i_{(s)}(t) = \sum_{n=1,3,5} \frac{4I_o}{n\pi} \sin(n\omega t - \alpha)$$

$$I_{s1} = \frac{2\sqrt{2}}{\pi} I_o$$

RMS value of total input current $I_s = I_o$

current distortion factor (CDF) = $\frac{I_{s1}}{I_s}$

Power factor (p.f.) = CDF × DF (Displacement factor)

$$\text{p.f.} = \frac{2\sqrt{2}}{\pi} \cos \alpha$$

Harmonic factor or total harmonic distortion

$$(\text{THD}) = \left[\frac{1}{(\text{CDF})^2} - 1 \right] = \left[\left(\frac{\pi}{2\sqrt{2}} \right)^2 - 1 \right] = 48.34\%$$

17. Avg output voltage

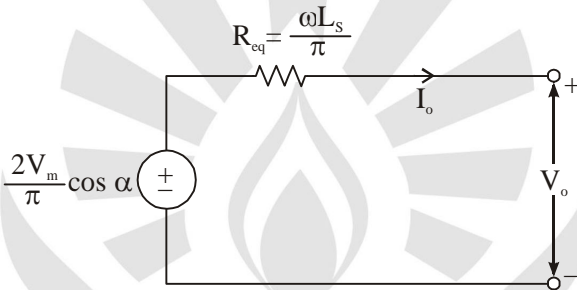
$$V_o = \frac{1}{\pi} \int_{\alpha+\mu}^{\alpha+\pi} V_m \sin \theta d\theta$$

$$V_o = \frac{V_m}{\pi} [\cos(\alpha + \mu) - \cos(\alpha - \pi)]$$

$$= \frac{V_m}{\pi} [\cos \alpha + \cos(\alpha + \mu)]$$

$$V_o = \frac{V_m}{\pi} \left[\cos \alpha + \left\{ \cos \alpha - \frac{\omega L_s I_o}{V_m} \right\} \right]$$

$$= \frac{2V_m \cos \alpha}{\pi} - \frac{\omega L_s I_o}{\pi}$$



So, due to L_s there is drop in the output voltage depending upon I_o the equivalent circuit of C_2 can be represented as voltage at no load

i.e. $I_o = 0$

$$V_{o(nl)} = \frac{2V_m \cos \alpha}{\pi}$$

18. When $\alpha < 60^\circ$

It behaves as a 6-pulse converter and least ripple frequency $f_o = 6f_s$ and FD does not conduct in this firing angle range.

When $\alpha \geq 60^\circ$

It behaves as a 3-pulse converter and least ripple frequency $f_o = 3f_s$, FD conducts during $(\alpha - 60^\circ)$ in each cycle of output.

$$\left. \begin{array}{l} \text{in both the cases} \\ \alpha < 60^\circ \text{ and } \alpha \geq 60^\circ \end{array} \right\}$$

$$V_{o \text{ avg}} = \frac{3V_{m1}}{2\pi} (1 + \cos \alpha)$$

19. Commutating capacitor (C) : Its value depends upon the turn-OFF time t_c to the main thyristor T_1 . During the time t_c , capacitor voltage changes linearly from $(-V_s)$ to zero mode-III, It is known that $i_c = \frac{CdV}{dt}$ for a constant load current I_o , the above relation can be

written as $I_c = \frac{CV_s}{t_s}$ or $C = \frac{t_c I_o}{V_s}$ the commutation circuit turn off time t_c must be greater than the thyristor turn-off time t_q .

$$\text{Let } t_c = t_q + \Delta t$$

$$C = \frac{(t_q + \Delta t)I_o}{V}$$

20. The main problem in 180° mode is that $\theta = \pi$, T_1 is commutated and T_4 is fired, both T_1 and T_4 may conduct simultaneously short duration after $\theta = \pi$, due to this overlapping T_1 and T_4 there is short circuit of supply.

To avoid this overlapping a dead band is introduced between $T_1 T_4$. Better to commutate T_1 some what earlier than $\theta = \pi$ thus in the dead band neither T_1 nor T_4 conduct. Hence for thyristor are mode to conduct for slightly less than 180° .

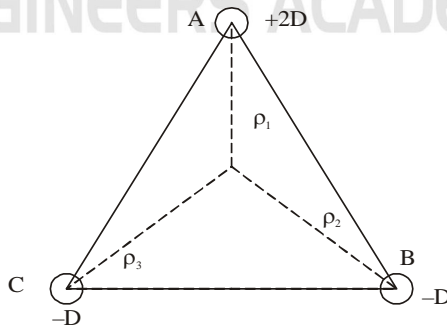
[PART : B]

21. Assuming the perpendicular distance from the equipotential point is ρ_1, ρ_2, ρ_3 from $+2D, -D, +D$ respectively.

Potential due to linear charge density at a distance ' ρ ' is given by

$$V = -\frac{\rho_L}{2\pi\epsilon_0} \ln(\rho)$$

Potential due to three charge density



$$V = V_1 + V_2 + V_3$$

$$= -\frac{2D}{2\pi\epsilon_0}\ln(\rho_1) + \frac{D}{2\pi\epsilon_0}\ln(\rho_2) + \frac{D}{2\pi\epsilon_0}\ln(\rho_3)$$

$$V = -\frac{D}{2\pi\epsilon_0} \left[\ln\left(\frac{\rho_1^2}{\rho_2 \cdot \rho_3}\right) \right] = \frac{D}{2\pi\epsilon_0} \ln\left(\frac{\rho_2 \cdot \rho_3}{\rho_1^2}\right)$$

To be V constant logarithmic term should be constant

$$\ln\left(\frac{\rho_2 \cdot \rho_3}{\rho_1^2}\right) = k$$

$$\frac{\rho_2 \cdot \rho_3}{\rho_1^2} = e^k$$

If the three wires are kept at corners at equilateral triangle then the distance from the centroid will be equal for all three.

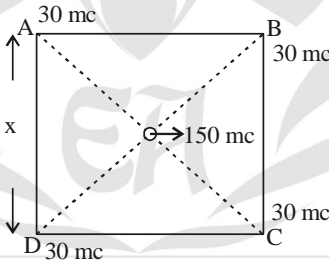
$$\rho_1 = \rho_2 = \rho_3 = k \text{ (constant)}$$

$$\therefore \ln\left(\frac{\rho_2 \rho_3}{\rho_1^2}\right) = \ln(1) = 0$$

Putting this value in equation of voltage, we get,

$$V = 0$$

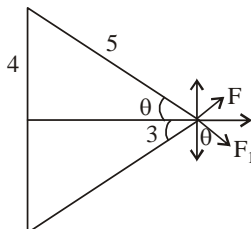
22.



$$AC = 8 = \sqrt{x^2 + x^2} = x\sqrt{2}$$

$$x = \frac{8}{\sqrt{2}}$$

$$OA = 4.$$



$$\cos\theta = 3/5$$

$$\begin{aligned}
 F_T &= 4F \cos\theta ; F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \\
 &= 4 \times \frac{30 \times 150 \times 10^{-12}}{4\pi\epsilon_0 \times 25} \cos\theta \\
 &= \frac{4 \times 45 \times 10^{-10} \times 9 \times 10^9}{25} \cos\theta \\
 &= \frac{4 \times 9 \times 0.9}{5} \cos\theta = 0.8 \times 8.1 \text{ N} \times \frac{3}{5} \\
 &= 6.48 \times 0.6 = 3.88 \text{ N}
 \end{aligned}$$

23. In Amplitude modulation

$$I_t = I_c \left(\sqrt{1 + \frac{\mu^2}{2}} \right)$$

I_t : AM Transmitter antenna current

I_c : Carrier current or AM modulated current

μ : Modulation index or depth of modulation

Here $I_t = 9 \text{ AmP}$, $I_c = 8 \text{ AmP}$,

$$\mu_2 = 0.5$$

By 1st modulation, modulation index is μ_1

$$I_t = I_c \sqrt{1 + \frac{\mu_1^2}{2}} \Rightarrow 9 = 8 \sqrt{1 + \frac{\mu_1^2}{2}}$$

or $\mu_1^2 = 0.53$ So $\mu_1 = 0.73$

If modulation index is changed to 0.5 then

$$I_t = 8 \left[1 + \frac{(0.5)^2}{2} \right]^{1/2}$$

$$I_t = 8 \left[1 + \frac{1}{8} \right]^{1/2}$$

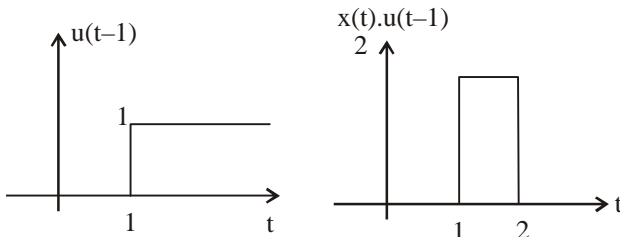
$$= 8 \sqrt{\frac{9}{8}} = \sqrt{72} \text{ AmP}$$

$$I_t = 8.48 \text{ Amp.}$$

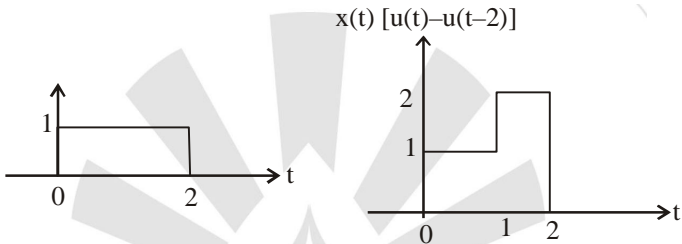
Hence if modulation index is decreased then transmitter current also decreases.

24. $x(t) u(t - 1)$

$u(t - 1)$ is the shifted step signal



(ii) $u(t) - u(t - 2)$ is a rectangular pulse



25. In communication system, the information is transmitted over a communication channel and received by receiver. For long distance transmission, modulation process is required.

Modulation :-

It is a process of translating low frequency message or information signal to a high frequency carrier signal, so that long distance transmission takes place.

Based on translating method of message signal to carrier signal, modulation can be defined as –

Modulation :

If one of the parameter of a carrier signal (Amplitude, frequency and phase) is varied according to message signal amplitude, that process is known as modulation.

Need of modulation :

1. To decide antenna height

The height of an antenna is dependent on the frequency that is to be transmitted. The antenna height is given by

$$h_t = \frac{\lambda}{4} \quad (\lambda : \text{wave length})$$

$$\lambda = \frac{C}{f} \quad (C : \text{light velocity ; } f : \text{frequency})$$

- For low frequency, antenna height is very high, which is practically not feasible

- For high frequency, antenna height is practically feasible.

2. For multiplexing

3. To convert low pass signal into Band pass signal

Demodulation :

It is a process of retrieving, original message signal from modulated signal

It can also be defined as

it is the process of converting band pass signal into base band or low pass signal.

Demodulation is employed at receiver.

26.

$$\vec{D} = D_0 \sin(\omega t + \beta z) \hat{a}_x \text{ for free space.}$$

$$B = ?$$

$$\vec{k} = |k| (-\hat{a}_z)$$

$$\vec{H} = \frac{1}{\eta_0} [\hat{k} \times \vec{E}] ; k = -\beta \hat{a}_z$$

$$D = \epsilon_0 \vec{E} \Rightarrow \vec{E} = \frac{D_0}{\epsilon_0} \sin(\omega t + \beta z) \hat{a}_x$$

$$\vec{H} = \frac{1}{\eta_0} [-\hat{a}_z \times \hat{a}_x] \frac{D_0}{\epsilon_0} \sin(\omega t + \beta z)$$

$$\vec{H} = \frac{1}{\sqrt{\frac{\mu_0}{\epsilon_0}}} \frac{D_0}{\epsilon_0} \sin(\omega t + \beta z) (-\hat{a}_y)$$

∴

$$\beta = \omega \sqrt{\mu_0 \epsilon_0}$$

$$\sqrt{\mu_0 \epsilon_0} = \frac{B}{\omega}$$

$$B = \mu_0 H = \frac{\mu_0 \omega D_0}{\beta} \sin(\omega t + \beta z) (-\hat{a}_y)$$

27. Switched mode power supply (SMPS):

Works like a chopper, by operating the on/off switch very rapidly, AC ripple frequency rises which can be easily filtered by L and C filter circuit which are small in size and less weighty. More over the voltage magnitude can be controlled by controlling the duty cycle of chopper. When SMPS operates at very high switching frequency we can reduce the size of filter as well transformer.

In SMPs transistor operates in switch mode (i.e., cut off region is used for off state and saturation region for ON state). The high speed devices SMPs is popularly used low a days. SMPs is more efficient and compact in Size.

28. When switch S is closed, KVL gives

$$Ri + \frac{1}{C} \int i dt = 0$$

Take laplace transform and initial condition.

$$RI_s + \frac{1}{C} \left[\frac{I_s}{s} - \frac{CV_o}{S} \right] = 0$$

$$\Rightarrow I_s = \frac{V_o}{s[R + 1/CS]}$$

Take inverse laplace

$$i(t) = \frac{V_o}{R} e^{-t/RC}$$

So, that peak diode current = $\frac{V_o}{R}$

$$\text{Capcitor voltage } V_C(t) = \frac{1}{C} \int_0^t i dt - V_o$$

$$= \frac{1}{C} \int_0^t \frac{V_o}{R} e^{-t/RC} .dt - V_o$$

$$V_C(t) = -V_o e^{-t/RC}$$

Energy dissipated in the circuit

$$= \frac{1}{2} CV_o^2 \text{ Joules.}$$

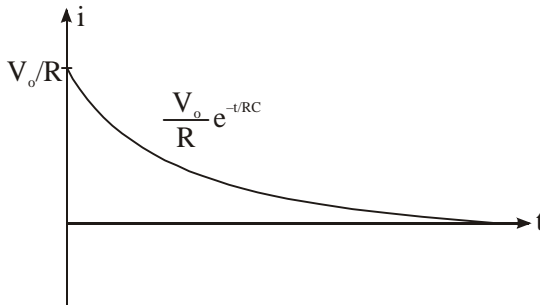


Figure : Wave form Discharging of Amp

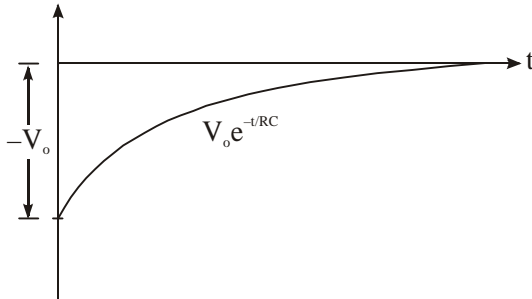
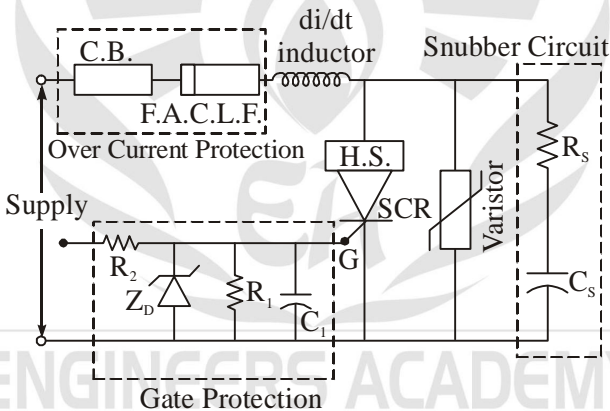


Figure : Wave form Charging of Voltage

29. Gate Protection :

Gate circuit should be protected against over voltage and over currents over voltages across the gate circuit can cause false triggering of the SCR over current may rises junction temp beyond specified limit leading to its damage. Protection against over voltages is achieved by connecting a zener diode Z_D across the gate circuit. A resistor R_2 connected in series with the gate circuit provides protection against over currents.

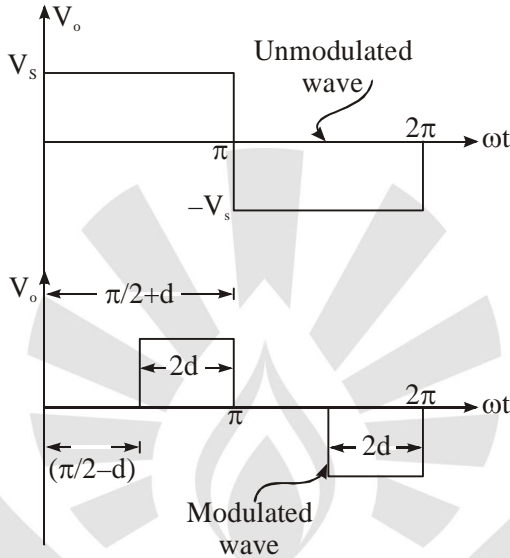


A resistor R_s is connected across gate-cathode terminal to bypass a part of thermally generated leakage current across junction J_2 (forward blocking mode) improves thermal stability of SCR, its $\frac{dv}{dt}$ rating, holding current and noise immunity of SCR.

30. Single pulse modulation (SPM) consist of a pulse of width $2d$ located symmetrically about $\frac{\pi}{2}$ and another pulse located symmetrically about $\frac{3\pi}{2}$.

The range of pulse-width $2d$ varies from 0 to π ; i.e. $0 < 2d < \pi$. The output voltage is controlled by varying the pulse width $2d$. This shape of the output voltage wave shown in figure is called quasi-square wave.

Total rms value of output voltage from figure



$$V_{or}^2 = \text{avg of } V_o^2 = \frac{1}{\pi} \int_0^{\pi} V_o^2 d\theta = \frac{1}{\pi} [V_s^2 \times 2d]$$

$$\Rightarrow V_{or} = V_s \sqrt{\frac{2d}{\pi}}$$

Fourier analysis of modulated wave shows that

$$V_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin \frac{n\pi}{2} \sin nd \sin n\omega t$$

$$V_o = \frac{4V_s}{\pi} \sin d \sin \omega t - \frac{4V_s}{3\pi} \sin 3d \sin 3\omega t + \frac{4V_s}{5\pi} \sin 5d \sin 5\omega t + \dots$$

When pulse width $2d$ is equal to its maximum value of π radius, then fundamental component of output voltage has a peak value

$$\text{of } V_{01m} = \frac{4V_s}{\pi}.$$

For pulse width $2d$, the peak value of fundamental component is

$$\frac{4V_s}{n\pi} \sin nd \text{ so, } n^{\text{th}} \text{ harmonic is reduced by a factor of } \sin nd.$$

To eliminate 3rd harmonic if $\sin 3d = 0$; $3d = 0, \pi, 2\pi, 3\pi, \dots$

If pulse width $2d = \frac{2\pi}{3} = 120^\circ$.

P.W. = $2d = 120^\circ$ to eliminate 3rd harmonic.

By controlling P.W. = $2d$, either voltage magnitude (V_{or}) can be controlled or harmonic can be eliminated but not the both simultaneously in the single pulse width modulation technique.

31. Figure shows the circuit of single phase full wave AC voltage controller having one SCR T_1 , the diode D_1, D_2, D_3 and D_4 are connected to form a bridge.

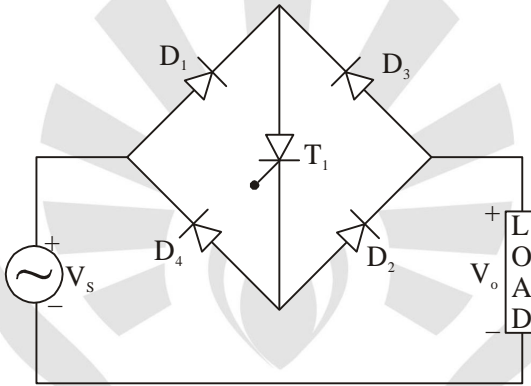


Figure: 1-f full wave controller with diode bridge and one SCR.

Operation :

- During positive half cycle, SCR T_1 is triggered at $\omega t = \pi$, load voltage and current reduces to zero and SCR T_1 is turned off due to natural commutation.
 - During negative half cycle, SCR T_1 is triggered at $\omega t = \pi + \alpha$. The current will flow through D_3, T_1, D_4 and the load. The load current and voltage becomes negative.
 - For this circuit, there is no need for any isolation between control and power circuits. This scheme, therefore offers a cheap AC voltage controller.
32. For constant load current I_o , current waveform for thyristor current i_T is as shown in figure.

$$I_o = \frac{V_o - E}{R}$$

The average thyristor current I_T is given by

$$I_T = I_o \frac{T_{ON}}{T} = \frac{V_o - E}{R} \alpha$$

$$I_T = \frac{\alpha V_s - E}{R} \cdot \alpha = \frac{\alpha^2 V_s - \alpha E}{R} \quad \dots(1)$$

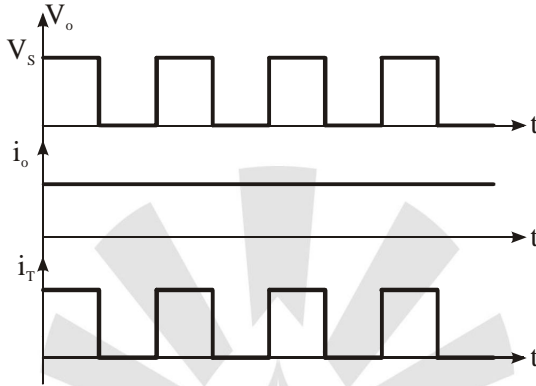


Figure: Wave form for Type-A Chopper

This will give a maximum value when

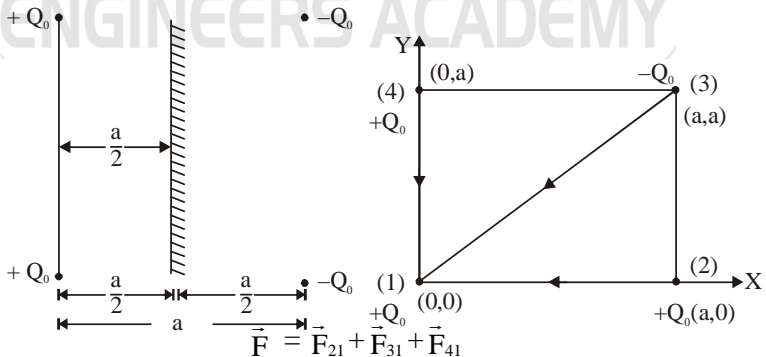
$$\frac{dI_T}{d\alpha} = \frac{2\alpha V_s - E}{R} = 0 \Rightarrow \alpha = \frac{E}{2V_s} \dots(2)$$

Therefore maximum value of average thyristor current is obtained by substituting the value of α from equation (2) in equation (1).

$$I_{Tmax} = \frac{E}{2V_s R} \left(\frac{E}{2V_s} \cdot V_s - E \right) = \frac{E^2}{4V_s R} \text{ Amp}$$

[PART : C]

33. (i)



We know that from coulomb's force law

$$\bar{F} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{|\bar{R}|^2} \frac{\bar{R}}{|\bar{R}|}$$

$$\bar{F}_{21} = \frac{-Q_0^2}{4\pi\epsilon_0 (a)^2} \frac{(-a\hat{a}_x)}{a} = \frac{Q_0^2}{4\pi\epsilon_0 a^2} \hat{a}_x$$

$$\bar{F}_{41} = \frac{Q_0^2}{4\pi\epsilon_0 (a)^2} \frac{(-a\hat{a}_y)}{a} = \frac{-Q_0^2}{4\pi\epsilon_0 a^2} \hat{a}_y$$

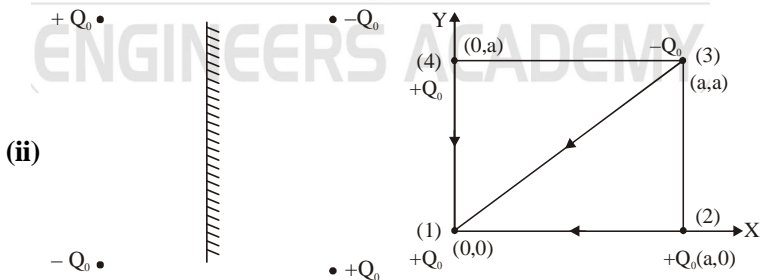
$$\begin{aligned} \bar{F}_{31} &= \frac{-Q_0^2}{4\pi\epsilon_0 (\sqrt{2}a)^2} \left(\frac{-a\hat{a}_x - a\hat{a}_y}{a\sqrt{2}} \right) \\ &= \frac{Q_0^2}{4\pi\epsilon_0 2\sqrt{2}a^2} [\hat{a}_x + \hat{a}_y] \end{aligned}$$

$$\bar{F} = \hat{a}_x \left[\frac{Q_0^2}{4\pi\epsilon_0 a^2} + \frac{Q_0^2}{4\pi\epsilon_0 2\sqrt{2}a^2} \right] + \hat{a}_y \left[\frac{Q_0^2}{4\pi\epsilon_0 2\sqrt{2}a^2} - \frac{Q_0^2}{4\pi\epsilon_0 a^2} \right]$$

$$= \frac{Q_0^2}{4\pi\epsilon_0 a^2} \left[\hat{a}_x \left(1 + \frac{1}{2\sqrt{2}} \right) + \hat{a}_y \left(-1 + \frac{1}{2\sqrt{2}} \right) \right]$$

$$\bar{F} = \frac{Q_0^2}{4\pi\epsilon_0 a^2} [1.35\hat{a}_x - 0.65\hat{a}_y]$$

$$\bar{F} = \frac{3Q_0^2}{8\pi\epsilon_0 a^2}$$



$$\bar{F}_{21} = \frac{-Q_0^2}{4\pi\epsilon_0 (a)^2} \left(-\frac{a\hat{a}_x}{a} \right) = \frac{-Q_0^2}{4\pi\epsilon_0 a^2} \hat{a}_x$$

$$\bar{F}_{41} = \frac{-Q_0^2}{4\pi\epsilon_0(a)^2} \left(-\frac{a\hat{a}_y}{a} \right) = \frac{-Q_0^2}{4\pi\epsilon_0 a^2} \hat{a}_y$$

$$\begin{aligned} \bar{F}_{31} &= \frac{-Q_0^2}{4\pi\epsilon_0(\sqrt{2}a)^2} \left(\frac{-a\hat{a}_x - a\hat{a}_y}{a\sqrt{2}} \right) \\ &= \frac{-Q_0^2}{4\pi\epsilon_0 2\sqrt{2}a^2} (\hat{a}_x + \hat{a}_y) \end{aligned}$$

$$\begin{aligned} \bar{F} &= \hat{a}_x \left[\frac{Q_0^2}{4\pi\epsilon_0 a^2} - \frac{Q_0^2}{4\pi\epsilon_0 a^2 2\sqrt{2}} \right] + \hat{a}_y \left[\frac{Q_0^2}{4\pi\epsilon_0 2\sqrt{2}a^2} + \frac{Q_0^2}{4\pi\epsilon_0 a^2} \right] \\ &= \frac{Q_0^2}{4\pi\epsilon_0 a^2} \left[\hat{a}_x \left(1 - \frac{1}{2\sqrt{2}} \right) + \hat{a}_y \left(\frac{-1}{2\sqrt{2}} + 1 \right) \right] \\ &= \frac{Q_0^2}{4\pi\epsilon_0 a^2} \{ 0.6464\hat{a}_x + 0.6464\hat{a}_y \} \\ &= \frac{0.914Q_0^2}{4\pi\epsilon_0 a^2} \end{aligned}$$

The force is decreasing when sign of the charge is reversed.

34. Given,

Unmodulated carrier power

$$P_C = 9 \text{ kW}$$

Modulated signal power

$$P_t = 10.125 \text{ kW.}$$

Let the modulation index be μ_1

Then total transmitted power is given by

$$P_t = P_C \left(1 + \frac{\mu_1^2}{2} \right)$$

$$\frac{P_t}{P_C} - 1 = \frac{\mu_1^2}{2}$$

⇒

$$2 \left(\frac{P_t}{P_C} - 1 \right) = \mu_1^2$$

$$\mu_1 = \sqrt{2 \left(\frac{P_t}{P_c} - 1 \right)}$$

$$\mu_1 = \sqrt{2 \left(\frac{10.125}{9} - 1 \right)} = \sqrt{2 \left(\frac{10.125 - 9}{9} \right)}$$

$$\mu_1 = 0.5$$

Another sine wave corresponding to 40% modulation is simultaneously transmitted.

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.5)^2 + (0.4)^2} = \sqrt{0.25 + 0.16} = 0.64$$

$$\mu_t = 0.64$$

The total radiated power is given by

$$\begin{aligned} P_{AM} &= P_c \left(1 + \frac{\mu_t^2}{2} \right) = 9 \left(1 + \frac{(0.64)^2}{2} \right) \\ &= 9 \left(1 + \frac{0.4096}{2} \right) = 9(1 + 0.2048) \\ &= 10.84 \text{ kW} \end{aligned}$$

$$P_{AM} = 10.84 \text{ kW.}$$

35. Given

$$f_{AM}(t) = 20 \cos(2\pi \times 10^6 t) + 10 (\cos 2\pi \times 10^6 t) \cos(2\pi \times 10^3 t) + y \cos(2\pi \times 10^6 t) \cos(4\pi \times 10^3 t)$$

$$f_{AM}(t) = 20 \left[1 + \frac{1}{2} \cos 2\pi \times 10^3 t + \frac{y}{20} \cos 4\pi \times 10^3 t \right] \cos 2\pi \times 10^6 t$$

A general AM signal is given by

$$f_{AM}(t) = A_c [1 + \mu_1 \cos 2\pi f_{m1} t + \mu_2 \cos 2\pi f_{m2} t] \cos 2\pi f_c t \quad \dots(2)$$

A multitone signal $m(t) = A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t$ used to amplitude modulate a carrier signal $c(t) = A_c \cos 2\pi f_c t$. then modulated signal has frequency components :

$$f_c, f_c + f_{m1}, f_c + f_{m2}, f_c - f_{m1}, f_c - f_{m2}$$

By equation (i) and (ii)

$$f_c = 10^6 \text{ Hz,}$$

$$f_{m1} = 10^3 \text{ Hz,}$$

$$f_{m2} = 2 \times 10^3 \text{ Hz}$$

Hence output has 1000 kHz

1001 kHz, 1002 kHz

999 kHz, 998 kHz

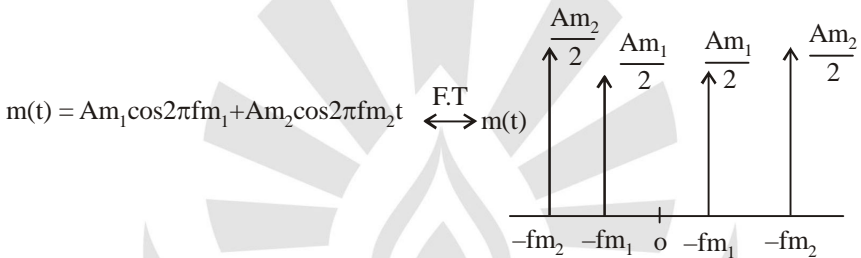
By comparison of equation (i) and (ii)

$$\text{Modulation index } \mu_1 = \frac{1}{2}$$

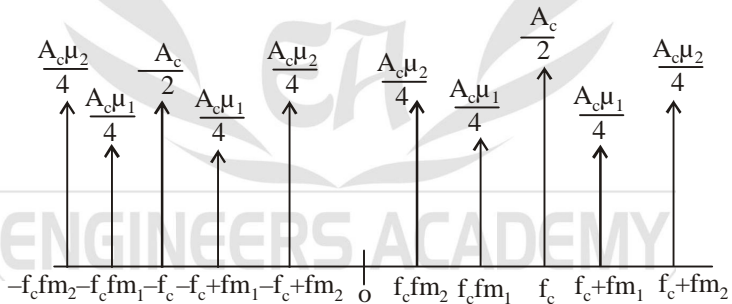
$$\text{Modulation index } \mu_2 = \frac{y}{20}$$

Spectrum :

$$m(t) = A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t$$



Then modulated spectrum



Bandwidth = maximum frequency component – Minimum frequency component

$$= (f_c + f_{m2}) - (f_c - f_{m2}) = 2 f_{m2}$$

$$f_{m2} = 2 \times 10^3 \text{ Hz}$$

Hence

$$\text{Bandwidth} = 2 \times 2 \times 10^3 = 4 \text{ kHz}$$

36. Maxwell's equation indifferently and intergal form derivations.

Differential form

$$\vec{\nabla} \times \vec{H} = J_C + J_d = \epsilon E + \frac{d}{dt} E \quad \dots(1)$$

Ampear's law

$$\oint \vec{H} \cdot d\vec{l} = I_{enc}$$

$$\oint_C (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \int (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \vec{J}_{eq} \cdot \vec{ds} \quad \dots(2)$$

By Continuity equation

$$q = i.t. \Rightarrow I = \frac{d}{dt} q$$

$$I = \frac{d}{dt} \int \vec{E} \cdot d\vec{s}$$

$$\int \vec{H} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \frac{d}{dt} \int E \cdot d\vec{s}$$

$$\nabla \times H = \frac{d}{dt} E \quad \dots(3)$$

Combining all 3 equation we get

$$\vec{\nabla} \times \vec{H} = \epsilon E + \frac{d}{dt} E$$

$$\vec{\nabla} \times \vec{E} = \mu \frac{dH}{dt} \quad \text{--Maxwell's 2nd Equation}$$

Faraday's Law :

$$V = -\frac{d}{dt} \int \vec{B} \cdot d\vec{s}$$

$$V = \int_C \vec{E} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{E}) \cdot d\vec{s}$$

$$= -\frac{d}{dt} \int \vec{B} \cdot d\vec{s}$$

$$\nabla \times E = -\mu \frac{dH}{dt}$$

Maxewell's IIIrd Equation :

$$\vec{\nabla} \cdot \vec{D} = P_v$$

According to Gams Low :

$$Q_{enc} = \oint_s \bar{D} \cdot \bar{ds}$$

$$\int_v P_v dv = \int_v (\bar{\nabla} \cdot \bar{D}) = \bar{\nabla} \cdot \bar{D} = \rho_v$$

Maxwell's -IV $\bar{\nabla} \cdot \bar{B} = 0$

Because in magnetic field no divergence loss occur.

Maxwell's equation in integral form

$$1. \quad \oint_s \bar{D} \cdot \bar{ds} = Q_{enc} = \text{Amount of charge in this surface } s.$$

$$2. \quad \oint_L \bar{E} \cdot \bar{dl} = - \int_s \frac{d}{dt} (\bar{B} \cdot \bar{ds})$$

$$3. \quad \oint_L \bar{H} \cdot \bar{dl} = I_{enc} + \iint_s \frac{d}{dt} \bar{D} \cdot \bar{ds} = I_{enc} = \iint_s \bar{J} \cdot \bar{ds}$$

$$4. \quad \iint_s \bar{B} \cdot \bar{ds} = 0$$

37. Switching, dynamic or transient char of SCR during turn-ON. The turn-ON time ' t_{ON} ' of the SCR is subdivided into three distinct period.

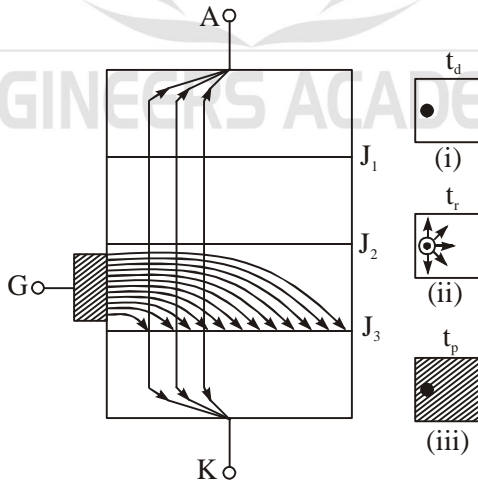
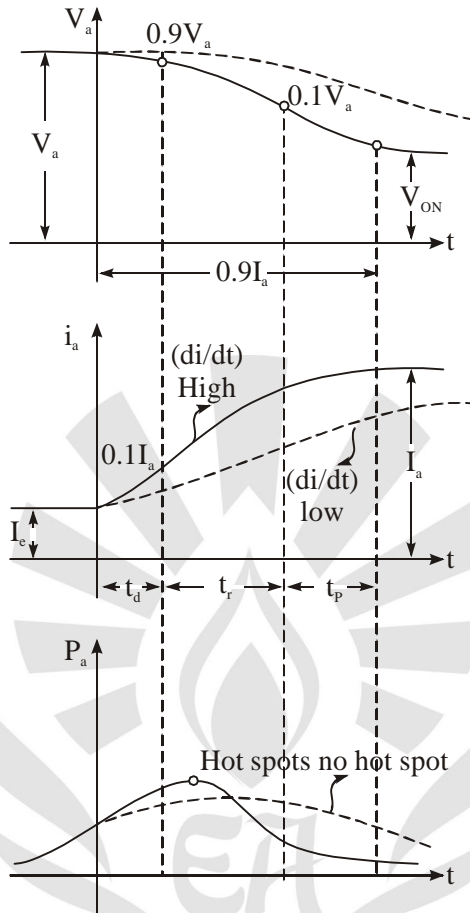
- (i) Delay time ' t_d '
- (ii) Rese Time ' t_r '
- (iii) Spread Time ' t_s '

• **Delay Time :**

This is the time between the instant at which the gate current reaches to 90% of it's final value (from 0 to 90% I_g) and the instant at which the anode current reaches 10% of its final value (i.e. 10% I_a)

It can also be defined as the time during which anode voltage fall from 100% of V_a to 90% of V_a as shown in figure.

During delay time gate current flows from gate to cathod has non-uniform distribution, more current density near gate but decreases rapidly as the distance from the gate increases hence anode current flows in a narrow region near gate os shown in figure.



The delay time can be decreases by applying high gate current and more forward voltage between anode to cathod. The delay time is fraction of a microsecond.

- **Rise Time :**

This is the time requires for anode current to rise from 10% to 90% of its final value.

It is also defined as the time during which anode voltage V_a falls from 90% of V_a to 10% V_a . The rise time is inversely proportional to the magnitude of gate current and its build up

rate i.e. $\frac{di_g}{dt}$, its depends on the nature of anode circuit. t_r can

be reduced if high and steep current pulses are applied to gate. During rise time anode current flows in narrow conducting area as turn-on losses in the thyristor are the highest due to high anode voltage and large anode current, occuring together in the thyristor. As these losses occur only over a small conducting region, loca hot spots may be formed and the device may be damaged i.e., $P = V_a I_a$ this power dissipation is called switching losses of thyristor.

- **Spread Time :**

Spread time is the time required for the anode voltage V_a to fall from 10% V_a to the ON-state voltage drop (1V to 1.5V) after spread time, anode current attains steady state values and the voltage drop across the SCR is equal to the ON-state voltage drop of the order of 1 V to 1.5 V during spread time the conduction spreads whole of cathode surface.

- **Switching Characteristics During Turn-OFF:**

Thyristor turn-off means that it has changed from ON to OFF state and is capable of blocking the forward voltage i.e. from conduction state to forward blocking state is called turn-off process. Thyristor can be turn-off by reducing the anode current below holding current i.e. ($I_a < I_H$). Still the device will not be able to block this forward voltage as the carriers (hole & electrons) in the four layers are favourable for conduction.

Turn-off time is divided into :

(i) Reverse recovery time ' t_{rr} '

(ii) Gate recovery time ' t_{gr} '

- **Reverse Recovery Time 't_{rr}'**

Once a mode current is zero, the device start to turn-OFF but not immediately and it will take some ot turn-OFF. The time taken by the minority carriers present in the PN-junction to recombine with opposite charges and to be neutralled this time is called reverse recovery time 't_{rr}'.

- **Gate recovery time 't_{gr}'**

The time taken by charges for the recombination when reverse voltage is maintanated across the thyristor. The time taken by the thyristor to change state from ON-state to OFF-state is called turn-OFF state. Turn-OFF time is the sum of t_{rr} and t_{gr} (t_{OFF} = t_{rr} + t_{gr})

38. Snubber circuit parameter R_s and C_s connected across SCR and $\frac{di}{dt}$ inductor L in series with anode circuit as shown in figure (b) when switch S is closed the cap behaviour like as a S/C and SCR in the forward blocking state offers a very high resistance. Therefore the equivalent circuit soon after the instant of closing the switch S is as shown in figure (c) for this circuit voltage equation is

$$V_s = (R_s + R_L)i + L \frac{di}{dt} \quad \dots(1)$$

It's solution given,

$$i = I(1 - e^{-t/\tau})$$

where $I = \frac{V_s}{R_s + R_L}$ and $\tau = \frac{L}{R_s + R_L}$

In equation (1) , t is the time in sec measured from the instant of closing the switch for this equation

$$\frac{di}{dt} = Ie^{-t/\tau} \cdot \frac{1}{\tau} = \frac{V_s}{R_s + R_L} \cdot \frac{R_s + R_L}{L} e^{-t/\tau}$$

$$\frac{di}{dt} = \frac{V_s}{L} e^{-t/\tau}$$

The value of $\frac{di}{dt}$ is maximum when t = 0

$$\therefore \left(\frac{di}{dt}\right)_{\max} = \frac{V_s}{L} \text{ or } L = \frac{V_s}{\left(\frac{di}{dt}\right)_{\max}} \quad \dots(2)$$

The voltage across SCR is given by,

$$V_a = R_s \cdot i \text{ or } \frac{dV_a}{dt} = R_s \cdot \frac{di}{dt}$$

$$\text{or } \left(\frac{dV_a}{dt} \right)_{\max} = R_s \cdot \left(\frac{di}{dt} \right)_{\max} \quad \dots(3)$$

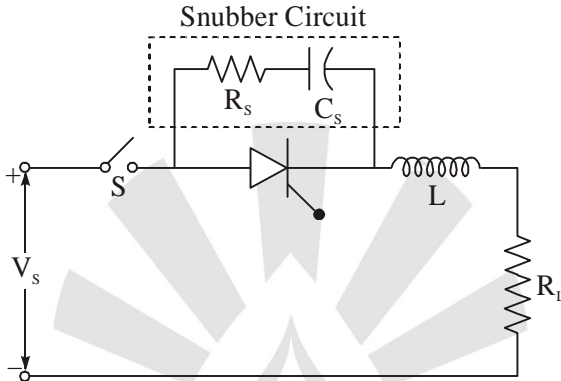


Figure : (b) Thyristor Protection
with \$L\$ and \$R_s\$, \$C_s\$

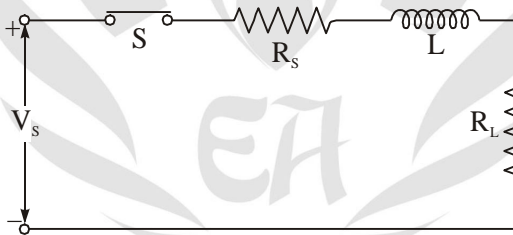


Figure : (c) Equivalent Circuit of

(b) the inst switchs is closed

From equation (2) and (3)

$$\left(\frac{dV_a}{dt} \right)_{\max} = \frac{R_s V_s}{L} \text{ or } R_s = \frac{L}{V_s} \left(\frac{dV_a}{dt} \right)_{\max} \quad \dots(4)$$

Analysis of this circuit shows that resistance \$R_s\$ can be obtained from second order characteristic equation \$s^2 + 2\xi\omega_n s + \omega_n^2 = 0\$ compar with series \$R, L, C\$ circuit equation

$$s^2 + \frac{(R_s)}{L}s + \frac{1}{LC_s} = 0$$

$$\frac{R_s}{L} = 2\xi \cdot \frac{1}{\sqrt{LC_s}} \Rightarrow R_s = 2\xi \sqrt{\frac{L}{C_s}}$$

where ξ is the damping factor (or damping ratio) in order to limit the peak voltage overshoot across thyristor to a safe value, damping factor in the range of 0.5 to 1 is usually used, for optimum solution of the problem ξ is taken to be about 0.65.

$$\therefore C_s = \frac{(2\xi)^2 L}{R_s} \text{ and } L \text{ from equation (4) } L = \frac{R_s V_s}{\left(\frac{dV_a}{dt}\right)_{\max}}$$

- 39.** To obtain four quadrant operation, without any mechanical change over switch, two full converter (1-phase or 3-phase) are connected back to back across the load such an arrangement using two-full converters in antiparallel and connected to the same DC load is called a dual converter.

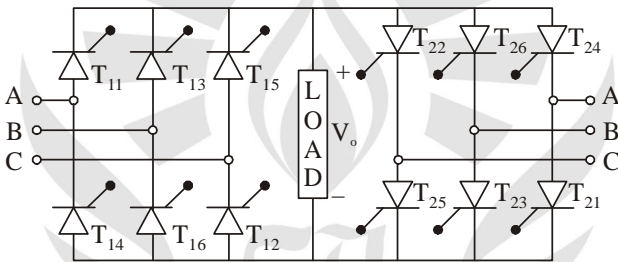


Figure : Non-Circulating Type
3-φ dual converter

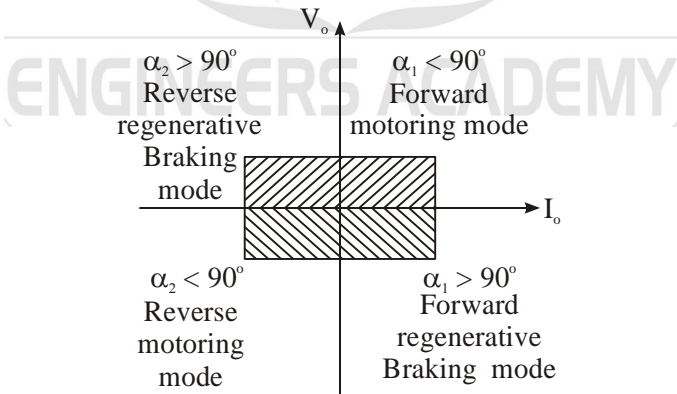


Figure : Four Quadrant
Operation Diagram

Ideal Dual Converter :

Assume that the dual converter consists of two ideal converter and there is no ripple in their output voltage i.e. if only DC values are considered.

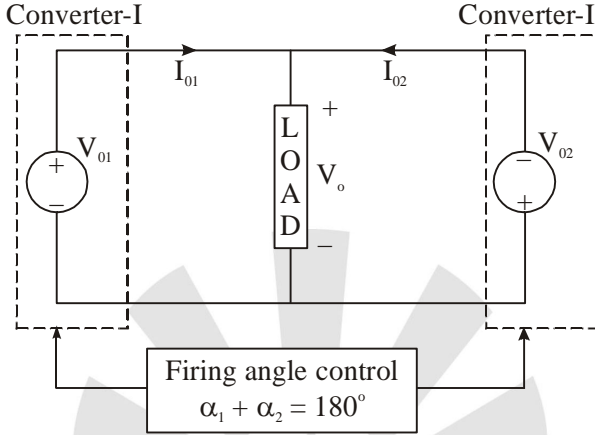


Figure : Equivalent circuit of
Ideal dual Converter

If firing angles of both the converters are controlled in such a manner that their avg output voltage are equal in magnitude and have the same polarity this can happen only if one converter is operating as a rectifier and other as an inverter, with the firing angles controlled in a manner that $\alpha_1 + \alpha_2 = 180^\circ$ and with both the converter in operation, their average output voltage are equal and have the same polarity.

There are two fundamental modes of a dual converter.

- **Non-Circulating Current Mode :**

Only one converter is in operation at a time and it alone carries the entire load current only this converter receives the firing pulses from the trigger control.

The Other Converter is Blocked From Conduction : This is achieved by removing the firing pulse from this converter. The two groups can not be operated simultaneously because it may result short circuit of the power supply. It is assumed that the two converters have their avg output voltages equal in magnitude their output voltages would have the same polarity only if polarity of V_{o2} is reversed to avoid DC circulating current

$$V_o = V_{o1} = V_{o2}$$

$$\frac{2V_m}{\pi} \cos \alpha_1 = -\frac{2V_m}{\pi} \cos \alpha_2 \quad (\text{in 1-phase})$$

$$\frac{3V_{m1}}{\pi} \cos \alpha_1 = -\frac{3V_{m1}}{\pi} \cos \alpha_2$$

$$\cos \alpha_1 = -\cos \alpha_2 \quad (\text{in 3-phase})$$

$$\cos \alpha_1 = \cos(180^\circ - \alpha_2)$$

$$\Rightarrow \alpha_1 + \alpha_2 = 180^\circ$$

The sum of firing angle of +ve and -ve group converters must be always

$$180^\circ \text{ i.e. } \alpha_1 + \alpha_2 = 180^\circ \text{ i.e., } \alpha_p + \alpha_n = 180^\circ$$

- **Dual Converter with Circulating Current :**

A reactor is inserted in between converter 1 and 2 as shown in figure this reactor limits the magnitude of circulating current to a reasonable value.

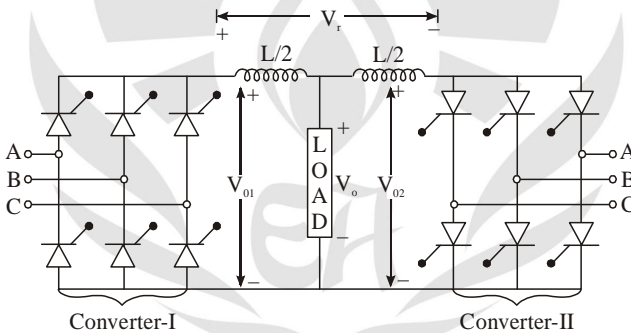


Figure: Circulating Current Type

Dual Converter for 3- ϕ Supply

To reduce the circulating current due to time an interphase reactor or mid point inductor is provided. It will act as a short circuit for DC. The normal delay period of 10 to 20 m sec, as required in circulating current free operation, is not needed here this makes the dual converter with circulating current operation faster.

As the converter have to handle load as well as circulating currents, the thyristor for the two converter are rated for higher currents.

